The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: November 16, 2017 Course: EE 313 Evans

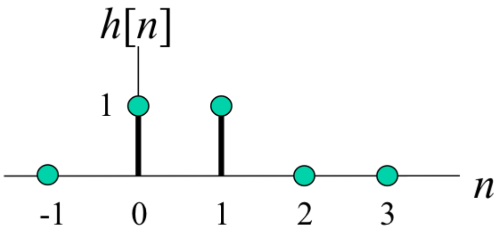
Name: ____
Last, First

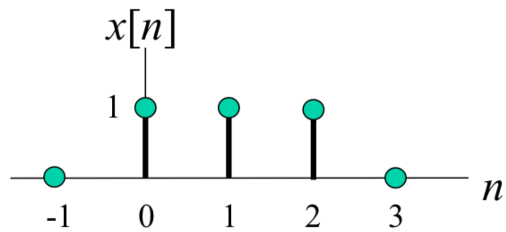
- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please turn off all cell phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	18		Discrete-Time Convolution
2	18		Continuous-Time Convolution
3	18		Discrete-Time First-Order System
4	24		Discrete-Time Second-Order System
5	22		Potpourri
Total	100		

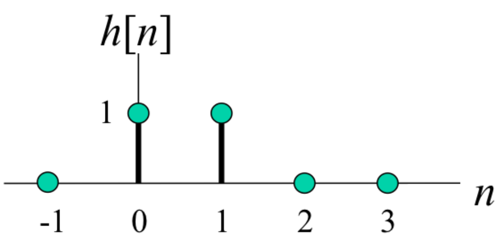
Problem 2.1 *Discrete-Time Convolution*. 18 points.

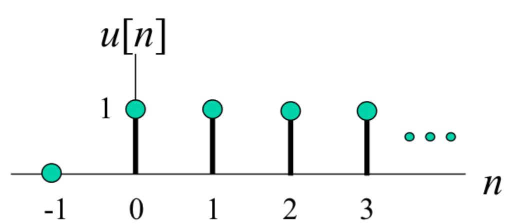
(a) Plot $y[n] = h[n] * x[n]$ using the rectangular pulse signals below. *9 points.*





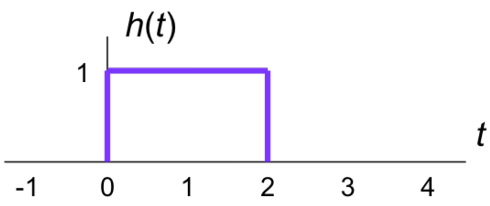
(b) Plot $y[n] = h[n] * u[n]$ using the signals below, where $h[n]$ is a rectangular pulse and $u[n]$ is the unit step signal. *9 points*.

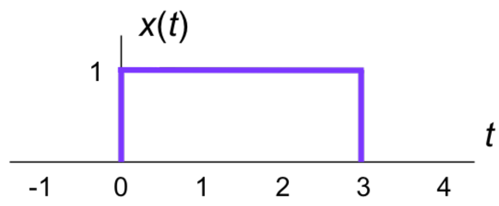




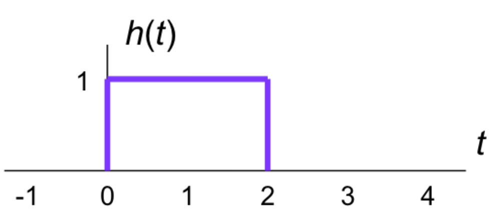
Problem 2.2 *Continuous-Time Convolution.* 18 points.

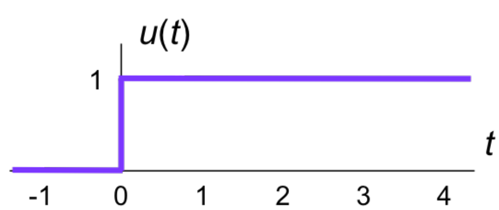
(a) Plot $y(t) = h(t) * x(t)$ using the rectangular pulse signals below. *9 points.*





(b) Plot $y(t) = h(t) * u(t)$ using the signals below, where $h(t)$ is a rectangular pulse and $u(t)$ is the unit step signal. *9 points*





Problem 2.3. *Discrete-Time First-Order LTI IIR System.* 18 points**.**

Consider a causal discrete-time first-order linear time-invariant (LTI) system with input $x[n]$ and output $y[n]$ governed by the following input-output relationship

$$y[n] - a\, y[n-1] = x[n] - b\, x[n-1]$$

for real-valued constants a and b where $|a| < 1$ and $|b| \geq 1$.

(a) Draw the block diagram for the input-output relationship in the discrete-time domain. *3 points.*

(b) What are the initial conditions? What should their values be? Why? *3 points.*

(c) Derive the transfer function in the z-domain. *3 points.*

(d) Give a formula for the frequency response. *3 points.*

(e) Give values of a and b to notch out a frequency of 0 rad/sample and pass other frequencies as much as possible. Justify your choices. *6 points.*

Problem 2.4 *Discrete-Time Second-Order LTI System.* 24 points.

The transfer function in the *z*-domain for a causal discrete-time second-order linear time-invariant (LTI) system is given below where $\hat{\omega}_0$ is a constant in units of rad/sample:

$$H(z) = \frac{(\sin\hat{\omega}_0)\ z^{-1}}{1 - 2(\cos\hat{\omega}_0)\ z^{-1} + z^{-2}}$$

(a) How many zeros are in the transfer function and what are their values? *3 points*.

(b) How many poles are in the transfer function and what are their values? *3 points*.

(c) What is the region of convergence? *3 points*.

(d) Derive the difference equation that relates input $x[n]$ and output $y[n]$ in the discrete-time domain. *6 points.*

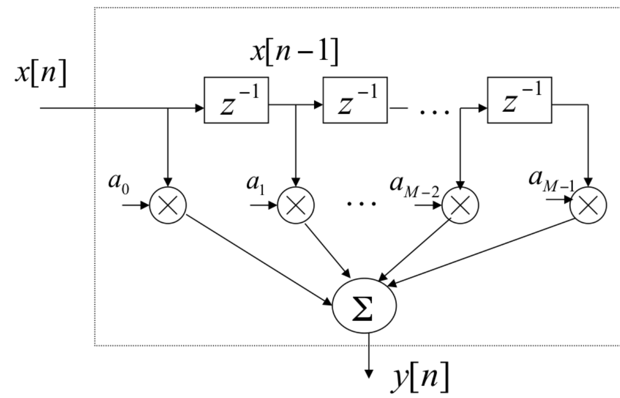
(e) What are the initial conditions? To what values should the initial conditions be set? *3 points*.

(f) Using the input-output relationship in part (d) and the initial conditions in part (e), compute the first three values of the impulse response for $n \geq 0$ to infer its formula. *Hint:* The impulse response is causal and periodic. *6 points*.

Problem 2.5. *Potpourri.* 22 points.

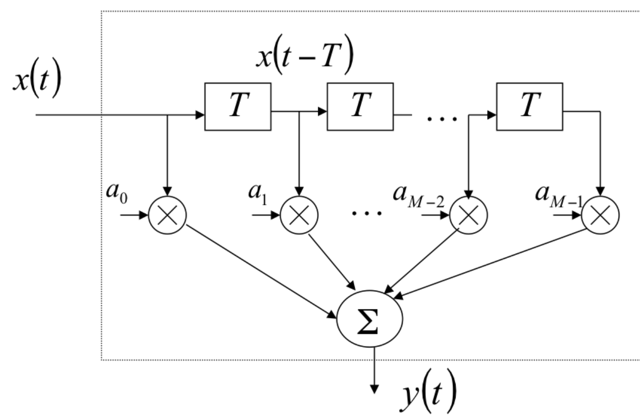
(a) Determine whether or not a tapped delay line is bounded-input bounded-output stability.

I. Discrete-time tapped delay line, a.k.a. finite impulse response filter. *6 points*.



$$y[n] = \sum_{k=0}^{M-1} a_k \; x[n-k]$$

II. Continuous-time tapped delay line. *6 points*.



$$y(t) = \sum_{k=0}^{M-1} a_k \; x(t-kT)$$

(b) Determine the number of coefficients of a discrete-time finite impulse response (FIR) averaging filter that would zero out 60 Hz and its harmonics. Use a sampling rate, f_s, of 480 Hz. *10 points.*